

# 

### Semester Two Examination, 2020

### Question/Answer booklet

# MATHEMATICS

**SOLUTIONS**

**APPLICATIONS**

**UNITS 3&4**

## Section One:

## Calculator-free

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| WA student number: In figures |  |  |  |  |  |  |  |  |  |  |

In words

Your name

|  |  |
| --- | --- |
| Number of additional answer booklets used (if applicable): |  |

## Time allowed for this section

Reading time before commencing work: five minutes

Working time: fifty minutes

## Materials required/recommended for this section

***To be provided by the supervisor***

This Question/Answer booklet

Formula sheet

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,  
correction fluid/tape, eraser, ruler, highlighters

Special items: nil

## Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of examination |
| Section One: Calculator-free | 8 | 8 | 50 | 52 | 35 |
| Section Two: Calculator-assumed | 13 | 13 | 100 | 98 | 65 |
|  | | |  | **Total** | 100 |

## Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen.  
Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

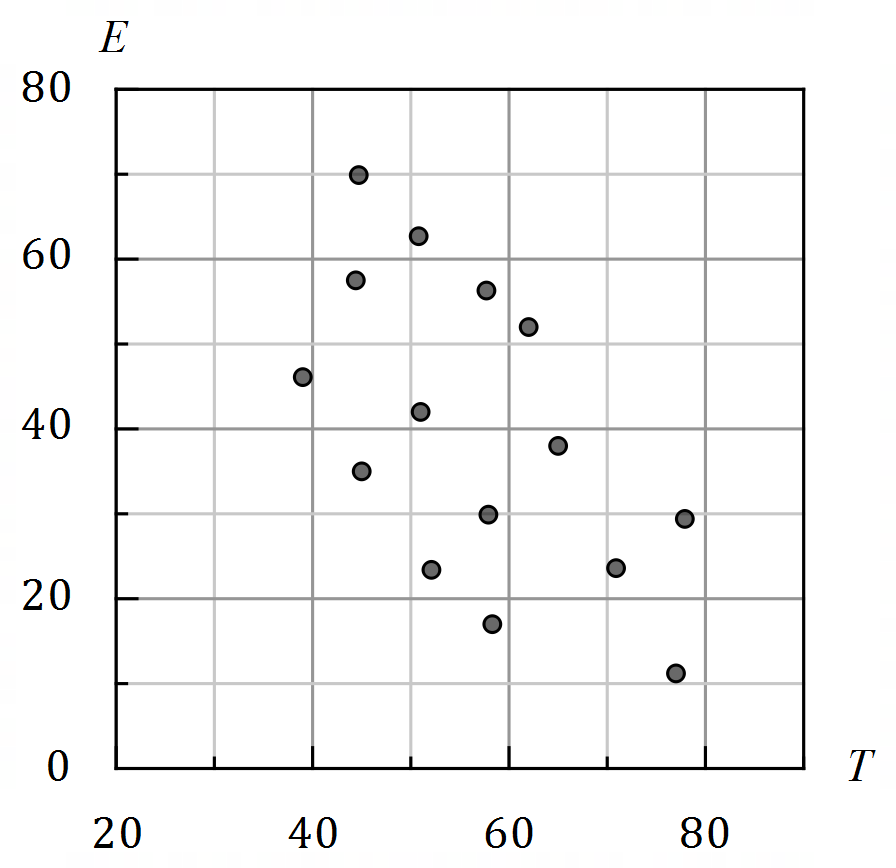
Section One: Calculator-free 35% (52 Marks)

This section has**eight** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 (5 marks)

A sample of fifteen people were asked to take between and minutes to practice a new skill and then their percentage error score in performing the skill recorded against their practice time minutes. The results are shown on the scatterplot below.



(a) Describe the association between and in terms of direction, form and strength.

(3 marks)

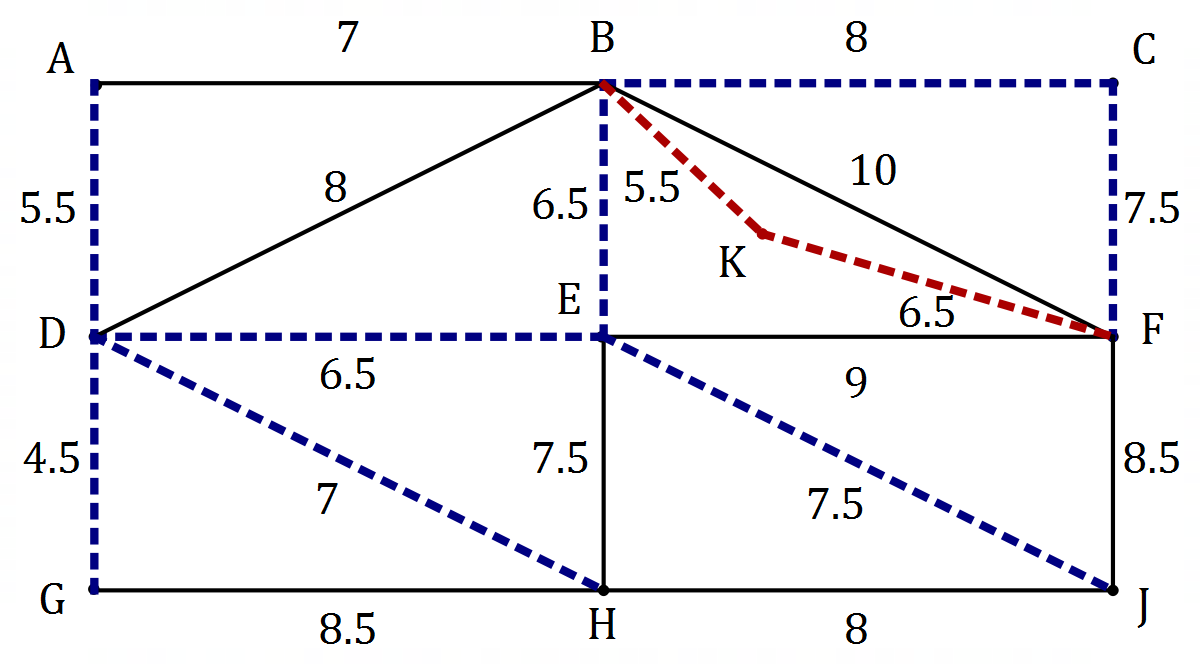
|  |
| --- |
| **Solution** |
| Direction is **negative**, form is **linear**, and strength is **moderate**. |
| **Specific behaviours** |
| ✓ direction  ü form  ü strength |

(b) Estimate, to one decimal place, the value of the correlation coefficient between the variables and hence determine the percentage of the variation in the error scores that can be explained by the variation in the practice times for this sample. (2 marks)

|  |
| --- |
| **Solution** |
| of the variation in can be explained by the variation in . |
| **Specific behaviours** |
| ✓ estimate to 1dp between and inclusive  ü correctly squares and writes as percentage |

Question 2 (7 marks)

The vertices on the graph below represent nine pumping stations. The edge weights are the number of hours required to install new electrical cabling between connected stations.



|  |
| --- |
| **Solution (a)** |
| See graph |
| **Specific behaviours** |
| ✓ any spanning tree  ü minimum spanning tree |

|  |
| --- |
| **Solution (c)(i)** |
| See graph |
| **Specific behaviours** |
| ✓ correctly adds edges, weights |

(a) Clearly show the minimum spanning tree on the graph. (2 marks)

A contractor charges per hour to install the cabling.

(b) Determine the cost to install new electrical cabling using the minimum spanning tree.

(2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ length of MST in hours  ü correct cost |

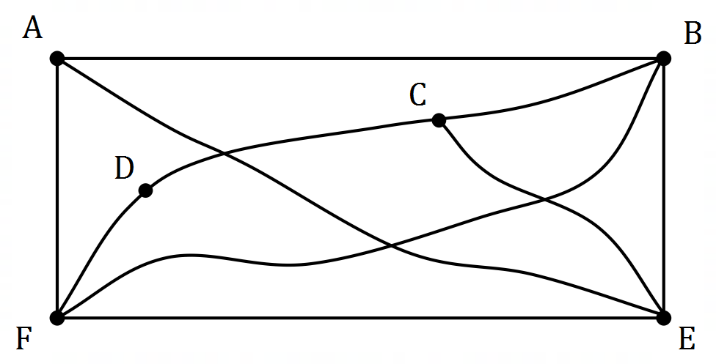
(c) A tenth pumping station is to be included. The time to install cable between and is hours and between and is hours.

(i) Use this information to add pumping station to the graph above. (1 mark)

(ii) If the new cabling is now installed using the minimum spanning tree that includes , determine the extra cost of the installation. (2 marks)

|  |
| --- |
| **Solution** |
| Changes to minimum spanning tree: add and , drop  Change to length of minimum spanning tree is .  Hence extra cost is . |
| **Specific behaviours** |
| ✓ indicates changes to minimum spanning tree  ü correct extra cost |

Question 3 (7 marks)

Graph is shown at right.

(a) Adding missing vertices as necessary to those below, draw graph in the plane, to clearly show that it is planar. (2 marks)

|  |
| --- |
| **Solution** |
| *(Example only)* |
| **Specific behaviours** |
| ✓ no edges that cross, at least correct vertex degrees  ü correctly drawn in the plane |

(b) Show that graph satisfies Euler's formula. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correctly counts  ü substitutes into Euler's formula and simplifies |

(c) Graph is semi-Eulerian. Describe two features of the trail it contains to be classified as semi-Eulerian. (2 marks)

|  |
| --- |
| **Solution** |
| The trail is open and includes every edge exactly once.  (*Only accept 'trail starts and ends at odd degree vertices' instead of 'trail is open' if answer makes clear the odd vertices are different.*) |
| **Specific behaviours** |
| ✓ indicates trail is open  ü indicates trail includes every edge exactly once |

(d) Describe where an edge can be added to graph so that it contains an Eulerian trail.

(1 mark)

|  |
| --- |
| **Solution** |
| Add edge between vertex and vertex . |
| **Specific behaviours** |
| ✓ indicates correct vertices |

Question 4 (6 marks)

Arif, Brenton, Carter and Dana have been chosen for the m medley relay team in a swimming carnival. This relay is swum by four different swimmers, each swimming one of four different strokes. Their best times, in seconds, to swim m freestyle, backstroke, breaststroke and butterfly are shown in the following table.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Freestyle | Backstroke | Breaststroke | Butterfly |
| Arif |  |  |  |  |
| Brenton |  |  |  |  |
| Carter |  |  |  |  |
| Dana |  |  |  |  |

(a) Show use of the Hungarian algorithm to determine which stroke each boy should swim so that the team completes the  m medley relay in the shortest possible time.

(4 marks)

|  |
| --- |
| **Solution** |
| Assignment: Arif - Butterfly; Brenton - Backstroke; Carter - Freestyle; Dana - Breastroke. |
| **Specific behaviours** |
| ✓ reduces rows  ü reduces columns  ü reduces again so that four lines needed to cover  ü states assignment |

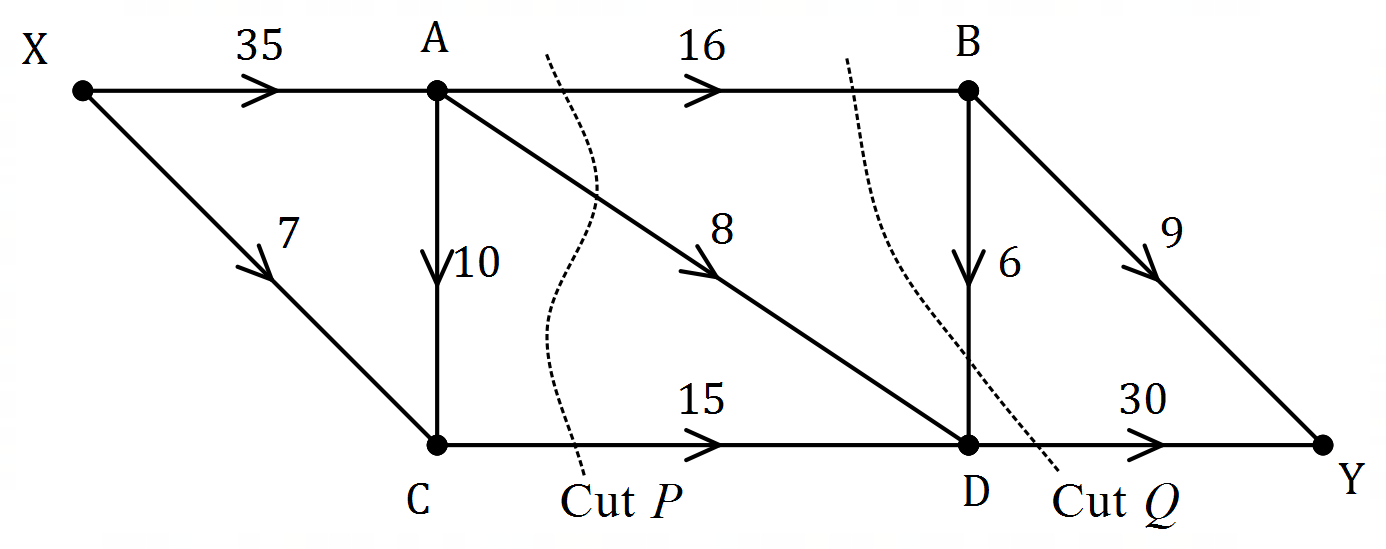
(b) The record for the m medley relay is minutes and seconds. If all the boys swim their assigned leg in their best time, will they break the record? Justify your answer.

(2 marks)

|  |
| --- |
| **Solution** |
| No. Their time will be seconds but the record is seconds, which is second quicker. |
| **Specific behaviours** |
| ✓ indicates time for boys  ü compares to record and states no |

Question 5 (8 marks)

A directed subgraph of a distribution network is shown below. The vertices represent distribution centres and the weight on each edge is the maximum volume of parcels, in cubic metres, that can be transported from one distribution centre to another every day.



(a) Determine the capacity of cut and the capacity of cut shown above. (2 marks)

|  |
| --- |
| **Solution** |
| Cut is .  Cut : . |
| **Specific behaviours** |
| ✓ cut ; ü cut |

(b) Determine the maximum volume of parcels that can be transported

|  |
| --- |
| **Solution** |
| Maximum volume is . |
| **Specific behaviours** |
| ✓ correct maximum flow |

(i) from centre X to centre C in a day. (1 mark)

(ii) from centre A to centre D in days. (2 marks)

|  |
| --- |
| **Solution** |
| Minimum cut is and so maximum volume in days is . |
| **Specific behaviours** |
| ✓ minimum cut; ü maximum volume |

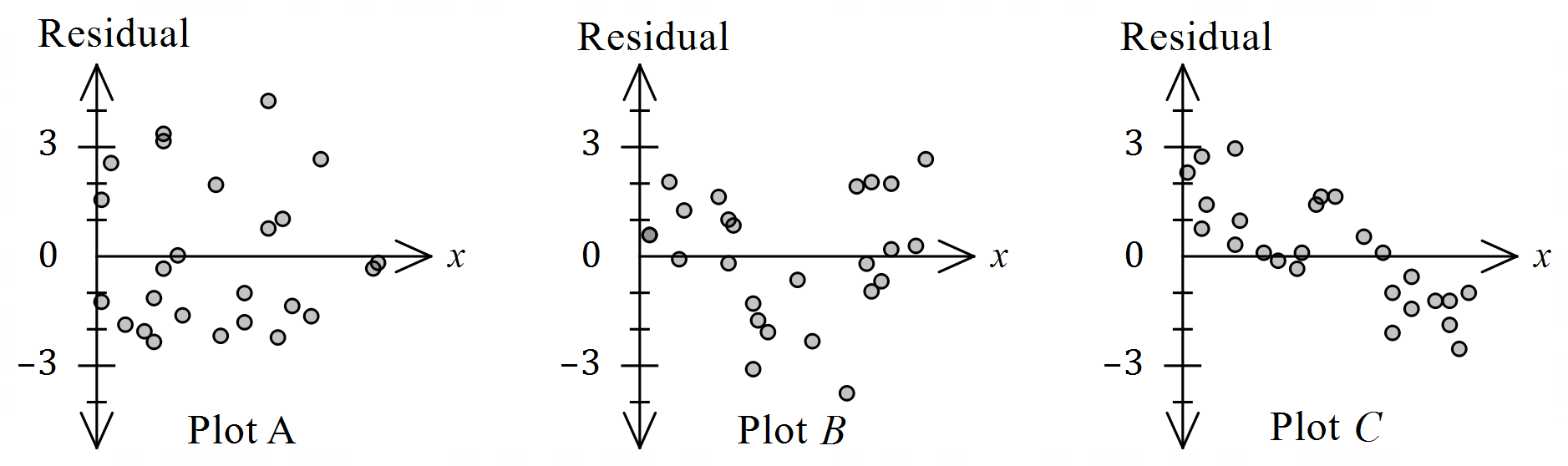
(c) Determine the maximum volume of parcels that can be transported from centre X to centre Y in days. (3 marks)

|  |
| --- |
| **Alternative Solution** |
| , , , , .  Maximum flow is .  Maximum volume in days is of parcels. |
| **Specific behaviours** |
| ✓ systematically lists flows  ü correct maximum flow  ü correct volume of parcels |

|  |
| --- |
| **Solution** |
| Cut , cut , cut  Minimum cut is .  Maximum volume in days is of parcels. |
| **Specific behaviours** |
| ✓ at least two more cuts  ü indicates minimum cut  ü correct volume of parcels |

Question 6 (6 marks)

A linear model was fitted to datasets and and the resulting residual plot for each model shown below. Dataset has a non-linear form whereas datasets and have linear form.



It is also known that the linear model was incorrectly fitted to dataset .

(a) State, with justification, which residual plot is most likely to be derived from dataset .

(2 marks)

|  |
| --- |
| **Solution** |
| Plot : A pattern is evident in the residual plot. (As increases, the residuals tend to be positive and then negative and then positive again.) |
| **Specific behaviours** |
| ✓ graph  ü justification |

(b) State, with justification, which residual plot is most likely to be derived from dataset .

(2 marks)

|  |
| --- |
| **Solution** |
| Plot : The residuals tend to be mostly positive for small and negative for large , which is unusual and suggests an incorrectly fitted linear model. |
| **Specific behaviours** |
| ✓ graph  ü justification |

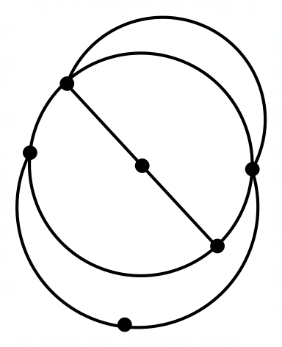
(c) State, with justification, which residual plot is most likely to be derived from dataset .

(2 marks)

|  |
| --- |
| **Solution** |
| Plot : There is no pattern evident in the residuals. |
| **Specific behaviours** |
| ✓ graph  ü justification |

|  |
| --- |
| **Note** |
| For one graph (correctly matched), accept the logical justification that it is the only dataset left - if that is the case. |

Question 7 (6 marks)

Graph is shown.

It represents a communication  
network of a group of six diplomats.

(a) State, with reasons, if graph is a simple graph. (2 marks)

|  |
| --- |
| **Solution** |
| No - it contains multiple edges. |
| **Specific behaviours** |
| ✓ states no  ü explanation |

(b) For graph , determine the length of the longest

(i) closed trail it contains. (1 mark)

|  |
| --- |
| **Solution** |
| Length is . |
| **Specific behaviours** |
| ✓ correct length |

(ii) open path it contains. (1 mark)

|  |
| --- |
| **Solution** |
| Length is . |
| **Specific behaviours** |
| ✓ correct length |

(c) Explain why graph is a Hamiltonian graph. (2 marks)

|  |
| --- |
| **Solution** |
| Graph contains a cycle (closed path) that includes each vertex in the graph. |
| **Specific behaviours** |
| ✓ indicates closed path  ü indicates path includes each vertex once only (apart from start/finish) |

Question 8 (7 marks)

A project involves the completion of activities to , as shown in the following table. Note that only three of the activity durations are shown.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Activity |  |  |  |  |  |  |  |  |
| Immediate predecessor(s) |  |  |  |  |  |  |  |  |
| Duration (weeks) |  |  |  |  |  |  |  |  |
| Float (weeks) |  |  |  |  |  |  |  |  |

The minimum completion time for the project is weeks.

(a) Construct an activity network to represent the above information. (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ shows most activities as labelled edges  ü correct predecessors for each activity, including dummy edge  ü correct activity network, with direction indicated |

(c) List the tasks that lie on the critical path. (1 mark)

|  |
| --- |
| **Solution** |
| The tasks are and (zero float). |
| **Specific behaviours** |
| ✓ correct tasks |

(b) Determine a possible duration for each of the activities and . (3 marks)

|  |
| --- |
| **Solution** |
| Using critical path: .  Using DurationLST of nextESTFloat:  Hence durations of and are and weeks respectively. |
| **Specific behaviours** |
| ✓ duration of  ü durations of and  ü durations of and |

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

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